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BUCKLING OF IMPERFECT SANDWICH CYLINDERS UNDER AXIAL COMPRESSION

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Abstract—An analytical study has been carried out to determine the effect of axisymmetric shape imperfections on the compressive buckling strength of sandwich cylinders having isotropic facings and orthotropic, shear deformable cores. Buckling solutions are presented as a function of imperfection amplitude, wavelength and the core shear flexibility coefficients. Non-shear deformable cores have also been considered and results compared to isotropic cylinders based on an "equivalent" thickness parameter.

INTRODUCTION

Circular cylindrical shell structures are widely used in aerospace vehicles, such as rockets, satellite components and aircraft fuselage sections. Although these structures are commonly fabricated from metals, advanced composites are also gaining widespread usage because of their higher strength/weight and stiffness/weight ratios. Prior to the advent of composites, however, the development of sandwich construction emerged because of the significant stiffness/weight improvements offered (Plantema, 1966). Most of the buckling analyses performed on sandwich cylinders have considered isotropic (Leggett and Hopkins, 1949; Teichmann *et al.*, 1951; Stein and Meyers, 1952; Wang *et al.*, 1955; Almroth, 1964; Barteld and Mayers, 1967) and orthotropic facings (March and Kuenzi, 1957; Reese and Bert, 1969, 1974; Zahn and Kuenza, 1963) with one recent report dealing with composite skins (Cheung and Tennyson, 1988). In all of these models, the core is regarded as "shear deformable" and the cylinders are treated as geometrically "perfect" in shape.

Up to this point in time, no Koiter-type imperfection analysis has been carried out on circular cylindrical sandwich cylinders under axial compression, to assess the extent to which these structures are sensitive to geometric shape imperfections. This report presents approximate analytical solutions for the buckling stress of axisymmetric imperfect sandwich cylinders with isotropic facings as a function of imperfection amplitude, wavelength and core shear stiffness. An "equivalent thickness" parameter is also defined that allows the design engineer to estimate "knock-down" factors for sandwich cylinders based on isotropic cylinder results, including the effect of random shape imperfections (Tennyson *et al.*, 1971).

FORMULATION OF PROBLEM

Shell configuration

The axisymmetric imperfect sandwich cylinder geometry and coordinate system are shown in Fig. 1. The cylinder geometry is characterized by its length L, radius of the median

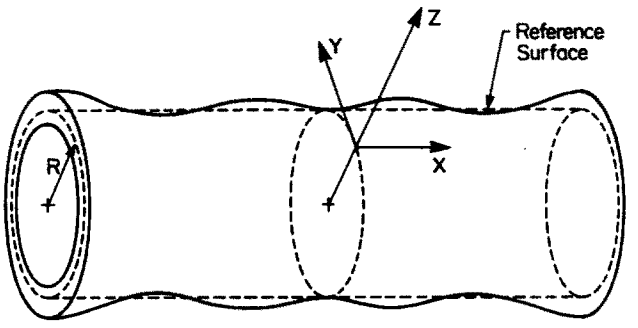

Fig. 1. Circular cylindrical shell with axisymmetric shape imperfections.

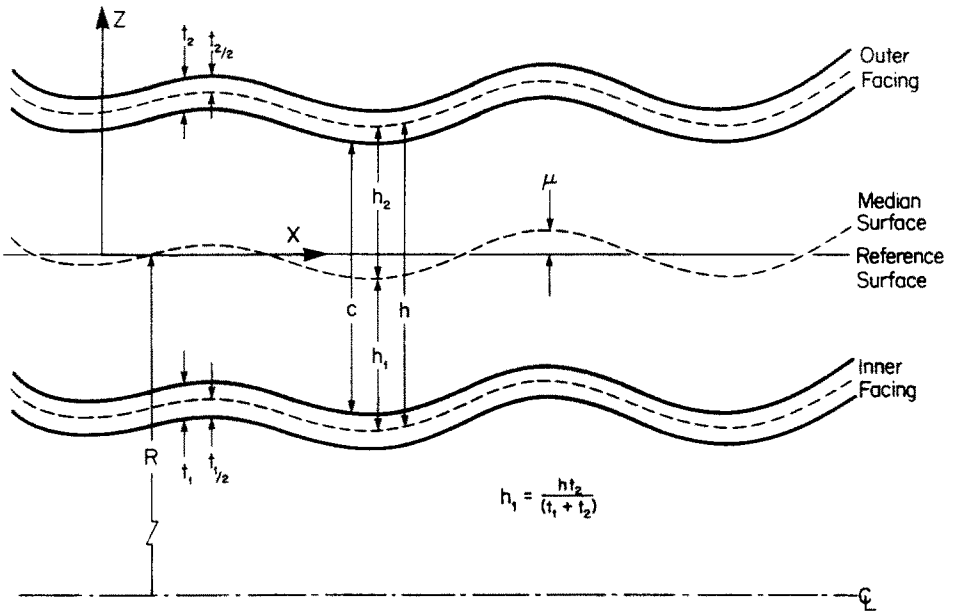

Fig. 2. Geometrical definition of sandwich shell wall.

surface R, inner face thickness t_1, outer face thickness t_2, core thickness c and imperfection amplitude μ (see Fig. 2).

Let the reference surface be the median surface of the geometrically perfect cylinder, as defined by:

$$h_1 = \frac{ht_2}{t_1 + t_2}. \tag{1}$$

Thus the separation between the mid-surfaces of the inner and outer facings is

$$h = c + \frac{t_1 + t_2}{2}. \tag{2}$$

The coordinate system, x, y, z is measured with respect to the reference surface in the axial, circumferential and radial directions, respectively. The components of displacement u, v and w of a point on the perfect shell are the displacements in the x, y and z directions.

Restrictions and assumptions

The following restrictions apply to the analytical model:

(1) Both facings are made of the same isotropic material.

(2) The core is orthotropic with one axis of orthotropy parallel to the axis of the cylinder.

(3) The core and facings have uniform elastic properties.

(4) Facings and core are of constant thickness throughout the shell wall. The two facings may have different thicknesses.

(5) There are no initial wrinkles in the facings, i.e. the separation between the facings is constant.

(6) There is no failure of bonding between facings and core.

(7) Facings are sufficiently thin (compared to the core) to be treated as membranes, i.e. the facings have in-plane stiffness but no flexural stiffness about their mid-surfaces.

(8) The shell thickness is small compared with the radius of curvature R.

(9) The cylinder is sufficiently long to ignore end boundary conditions.

(10) There is no intercell buckling.

The following assumptions are made in the analysis:

(1) Displacements u, v, and w are small compared with the plate thickness.
(2) Strains ε_x, ε_y and γ_{xy} are small compared to unity (small strain theory).
(3) The core carries no in-plane stress.
(4) Normal stiffness of the core is infinite so that instability associated with wrinkling of facings and other normal strain effects is not included. In practice, sandwich cylinders with honeycomb core will not fail by wrinkling of facings when there is no failure of bonding between facings and core.
(5) The transverse normal stress is negligible, i.e. $\sigma_z = 0$.

EQUILIBRIUM AND COMPATIBILITY EQUATIONS

Strain–displacement relations

Using the nonlinear Karman–Donnell strain–displacement relations, unit strains at a point in the shell can be expressed as,

$$\varepsilon_x = \frac{\partial \bar{u}}{\partial x} + \frac{1}{2}\left(\frac{\partial \bar{w}}{\partial x}\right)^2, \quad \varepsilon_y = \frac{\partial \bar{v}}{\partial y} + \frac{\bar{w}}{R} + \frac{1}{2}\left(\frac{\partial \bar{w}}{\partial y}\right)^2, \quad \varepsilon_z = 0$$

$$\gamma_{xy} = \frac{\partial \bar{v}}{\partial x} + \frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{w}}{\partial x}\frac{\partial \bar{w}}{\partial y}, \quad \gamma_{yz} = \frac{\partial \bar{w}}{\partial y} + \frac{\partial \bar{v}}{\partial z}, \quad \gamma_{xz} = \frac{\partial \bar{u}}{\partial z} + \frac{\partial \bar{w}}{\partial x}. \tag{3}$$

The condition of zero normal strain is the result of the assumption that the transverse normal stiffness is infinite. This also implies that $\bar{w}$ is not a function of z. For thin shells, $\bar{u}$ and $\bar{v}$ may be assumed to vary linearly in the z direction. The relations between the displacement components u, v and w of the deformed median surface and the displacement components $\bar{u}$, $\bar{v}$ and $\bar{w}$ of a point in the shell are:

$$\bar{u} = u - z\beta_x, \quad \bar{v} = v - z\beta_y, \quad \bar{w} = w, \tag{4}$$

where β_x and β_y can be interpreted physically as the components of change of slope of the normal to the undeformed median surface.

When the thickness of the facings is sufficiently small compared with the core, and when the transverse core shear strain is small, the strains for the mid-surfaces of the facings can be approximated by the strains at the facing and core interface. In this assumption, the facings are effectively considered to be membranes.

Thus by substituting eqn (4) into eqn (3) the strain–displacement relations become,

$$\varepsilon_x = \varepsilon_x^0 - z\frac{\partial \beta_x}{\partial x}, \quad \varepsilon_y = \varepsilon_y^0 - z\frac{\partial \beta_y}{\partial y}, \quad \gamma_{xy} = \gamma_{xy}^0 - z\left(\frac{\partial \beta_y}{\partial x} + \frac{\partial \beta_x}{\partial y}\right), \quad \gamma_{yz} = \gamma_{yz}^0, \quad \gamma_{xz} = \gamma_{xz}^0, \tag{5}$$

where the median surface strains ε_x^0, ε_y^0, γ_{xy}^0, γ_{yz}^0 and γ_{xz}^0 are given by:

$$\varepsilon_x^0 = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \quad \varepsilon_y^0 = \frac{\partial v}{\partial y} + \frac{w}{R} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2,$$

$$\gamma_{xy}^0 = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}, \quad \gamma_{yz}^0 = \frac{\partial w}{\partial y} - \beta_y, \quad \gamma_{xz}^0 = \frac{\partial w}{\partial x} - \beta_x \tag{6}$$

assuming the median surface displacements u, v do not vary in the z direction. The initial stress-free lateral deviation w of the median surface is assumed to be small, but finite values are also permissible provided that:

$$|w_0| \ll R \quad \text{and} \quad \left|\frac{\partial w_0}{\partial x}\right| \ll 1. \tag{7}$$

Shallow shell theory requires that the radius of curvature of the shape imperfection cannot be excessively small as expressed by;

$$R\left|\frac{\partial^2 w_0}{\partial x^2}\right| \leqslant 0(1). \tag{8}$$

The modified strain–displacement relations for the median surface including the initial deviation term w_0 are:

$$\varepsilon_x^0 = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{\partial w}{\partial x}\frac{\partial w_0}{\partial x}, \quad \varepsilon_y^0 = \frac{\partial v}{\partial y} + \frac{w}{R} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + \frac{\partial w}{\partial y}\frac{\partial w_0}{\partial y},$$

$$\gamma_{xy}^0 = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} + \frac{\partial w_0}{\partial x}\frac{\partial w}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w_0}{\partial y},$$

$$\gamma_{yz}^0 = \frac{\partial w}{\partial y} - \beta_y, \quad \gamma_{xz}^0 = \frac{\partial w}{\partial x} - \beta_x. \tag{9}$$

STRESS–STRAIN RELATIONS

Facings

The in-plane stress–strain relations for the isotropic facings in a plane stress state are:

$$\sigma_x = \frac{E_f}{(1-\nu^2)}(\varepsilon_x + \nu\varepsilon_y), \quad \sigma_y = \frac{E_f}{(1-\nu^2)}(\varepsilon_y + \nu\varepsilon_x), \quad \tau_{xy} = \frac{E_f}{2(1+\nu)}\gamma_{xy} \tag{10}$$

where E_f = modulus of elasticity of facings.

Stress–strain relations in terms of β_x, β_y and the median surface strains ε_x^0, ε_y^0 and γ_{xy}^0 can be obtained by substituting eqn (5) into eqn (10):

$$\sigma_x = \frac{E_f}{(1-\nu^2)}\left[\varepsilon_x^0 + \nu\varepsilon_y^0 - z\left(\frac{\partial \beta_x}{\partial x} + \nu\frac{\partial \beta_y}{\partial y}\right)\right]$$

$$\sigma_y = \frac{E_f}{(1-\nu^2)}\left[\varepsilon_y^0 + \nu\varepsilon_x^0 - z\left(\frac{\partial \beta_y}{\partial y} + \nu\frac{\partial \beta_x}{\partial x}\right)\right]$$

$$\tau_{xy} = \frac{E_f}{(1-\nu^2)}\left[\gamma_{xy}^0 - z\left(\frac{\partial \beta_y}{\partial x} + \frac{\partial \beta_x}{\partial y}\right)\right]. \tag{11}$$

Core

Assuming the core resists only transverse shear and does not carry any in-plane stresses, then the stress–strain relations for the core are:

$$\tau_{xz} = G_x\gamma_{xz} = G_x\gamma_{xz}^0, \quad \tau_{yz} = G_y\gamma_{yz} = G_y\gamma_{yz}^0, \quad \sigma_x = \sigma_y = \tau_{xy} = 0. \tag{12}$$

Shell forces and moments

An equivalent system of force and moment resultants is considered to be acting at the median surface of an element of the shell, as given by the following definitions:

Stress resultants:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \int_{-h_1-t_1/2}^{h_2+t_2/2} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} dz. \tag{13}$$

Transverse shear stress resultants:

$$\begin{bmatrix} Q_x \\ Q_y \end{bmatrix} = \int_{-h_1-t_1/2}^{h_2+t_2/2} \begin{bmatrix} \tau_{xz} \\ \tau_{yz} \end{bmatrix} dz. \tag{14}$$

Moment resultants:

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \int_{-h_1-t_1/2}^{h_2+t_2/2} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} z\, dz \tag{15}$$

where the integration is taken across the whole shell wall and h_1, h_2 are the distances of the middle surfaces of the inner and outer facings, respectively, from the median surface of the total shell wall as depicted in Fig. 2.

After performing the integration in eqn (13), using the expressions for the in-plane stresses σ_x, σ_y and τ_{xy} from eqn (11), the in-plane stress resultants in terms of β_x, β_y and the median surface strains are:

$$\begin{aligned} N_x &= \frac{E_f}{(1-\nu^2)}\left[(t_1+t_2)(\varepsilon_x^0+\nu\varepsilon_y^0)-(h_2t_2-h_1t_1)\left(\frac{\partial\beta_x}{\partial x}+\nu\frac{\partial\beta_y}{\partial y}\right)\right] \\ N_y &= \frac{E_f}{(1-\nu^2)}\left[(t_1+t_2)(\varepsilon_y^0+\nu\varepsilon_x^0)-(h_2t_2-h_1t_1)\left(\frac{\partial\beta_y}{\partial x}+\nu\frac{\partial\beta_x}{\partial y}\right)\right] \\ N_{xy} &= \frac{E_f}{(1-\nu^2)}\left[\gamma_{xy}^0-(h_2t_2-h_1t_1)\left(\frac{\partial\beta_y}{\partial x}+\frac{\partial\beta_x}{\partial y}\right)\right]. \end{aligned} \tag{16}$$

The transverse shear stresses τ_{yz} and τ_{xz} in the facings are equal to the transverse shear stresses in the core at the facings, and vanish on the free surfaces. Assuming a linear variation of transverse shear stresses across the facings and using eqn (12) for the transverse shear stresses in the core, the transverse shear stress resultants, after integration of eqn (14), become:

$$Q_x = G_x h\gamma_{xz}^0 = G_x h\left(\frac{\partial w}{\partial x}-\beta_x\right), \quad Q_y = G_y h\gamma_{yz}^0 = G_y h\left(\frac{\partial w}{\partial y}-\beta_y\right). \tag{17}$$

Again, using expressions for in-plane stresses from eqn (11), the moment resultants, after integration of eqn (15), become:

$$\begin{aligned} M_x &= \frac{E_f}{(1-\nu^2)}\left[(h_2t_2-h_1t_1)(\varepsilon_x^0+\nu\varepsilon_y^0)-\left(h_2^2t_2+\frac{t_2^3}{12}+h_1^2t_1+\frac{t_1^3}{12}\right)\left(\frac{\partial\beta_x}{\partial x}+\nu\frac{\partial\beta_y}{\partial y}\right)\right] \\ M_y &= \frac{E_f}{(1-\nu^2)}\left[(h_2t_2-h_1t_1)(\varepsilon_y^0+\nu\varepsilon_x^0)-\left(h_2^2t_2+\frac{t_2^3}{12}+h_1^2t_1+\frac{t_1^3}{12}\right)\left(\frac{\partial\beta_x}{\partial x}+\nu\frac{\partial\beta_y}{\partial y}\right)\right] \\ M_{xy} &= \frac{E_f}{(1-\nu^2)}\left[(h_2t_2-h_1t_1)\gamma_{xy}^0-\left(h_2^2t_2^2+\frac{t_2^3}{12}+h_1^2t_1+\frac{t_1^3}{12}\right)\left(\frac{\partial\beta_y}{\partial x}+\frac{\partial\beta_x}{\partial y}\right)\right]. \end{aligned} \tag{18}$$

With the definition of the median surface given by eqn (1), the coupling between in-plane strains and the rotation of the normal disappears, and the stress and moment resultants simplify to:

$$N_x = \frac{A}{(1-\nu^2)}(\varepsilon_x^0 + \nu\varepsilon_y^0), \quad N_y = \frac{A}{(1-\nu^2)}(\varepsilon_y^0 + \nu\varepsilon_x^0), \quad N_{xy} = \frac{A}{2(1+\nu)}\gamma_{xy}^0 \tag{19}$$

$$M_x = -D\left(\frac{\partial\beta_x}{\partial x} + \nu\frac{\partial\beta_y}{\partial y}\right), \quad M_y = -D\left(\frac{\partial\beta_y}{\partial y} + \nu\frac{\partial\beta_x}{\partial x}\right), \quad M_{xy} = \frac{-D(1-\nu)}{2}\left(\frac{\partial\beta_y}{\partial x} + \frac{\partial\beta_x}{\partial y}\right), \tag{20}$$

where the in-plane stiffness (A) and bending stiffness (D) parameters are given by:

$$A = E_f(t_1 + t_2) \tag{21}$$

$$D = \frac{E_f}{(1-\nu^2)}\left[\frac{h^2 t_1 t_2}{t_1 + t_2} + \tfrac{1}{12}(t_1^3 + t_2^3)\right]$$

$$\approx \left(\frac{E_f h^2 t_1 t_2}{(1-\nu^2)(t_1+t_2)}\right) \quad \text{for } t_1, t_2 \ll c. \tag{22}$$

Equilibrium equations

When the transverse normal stiffness of the sandwich shell is infinite, the equations of force and moment equilibrium are the same as those for conventional cylindrical shells. For a thin circular cylindrical shell the equilibrium equations are:

Equilibrium of horizontal forces:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0, \quad \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0. \tag{23}$$

Equilibrium of vertical forces:

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + N_x\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w_0}{\partial x^2}\right) + 2N_{xy}\left(\frac{\partial^2 w}{\partial x\,\partial y} + \frac{\partial^2 w_0}{\partial x\,\partial y}\right) + N_y\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w_0}{\partial y^2} - \frac{1}{R}\right) = 0. \tag{24}$$

Equilibrium of moments:

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} = Q_x, \quad \frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} = Q_y. \tag{25}$$

An Airy stress function is defined such that it will satisfy the equilibrium equations [eqn (23)] identically, i.e.

$$N_x = \frac{\partial^2 F}{\partial y^2}, \quad N_y = \frac{\partial^2 F}{\partial x^2}, \quad N_{xy} = -\frac{\partial^2 F}{\partial x\,\partial y}. \tag{26}$$

By substituting for M_x, M_y and M_{xy} from eqn (20) into eqn (25), Q_x and Q_y can be expressed in terms of β_x and β_y as:

$$Q_x = -D\left[\frac{\partial^2\beta_x}{\partial x^2} + \frac{(1+\nu)}{2}\frac{\partial^2\beta_y}{\partial x\,\partial y} + \frac{(1-\nu)}{2}\frac{\partial^2\beta_x}{\partial y^2}\right]$$

$$Q_y = -D\left[\frac{\partial^2\beta_y}{\partial y^2} + \frac{(1+\nu)}{2}\frac{\partial^2\beta_x}{\partial x\,\partial y} + \frac{(1-\nu)}{2}\frac{\partial^2\beta_y}{\partial x^2}\right]. \tag{27}$$

Replacing Q_x and Q_y in the above equations by expressions from eqn (17), the relations between w and β_x, β_y are:

$$\beta_x - \frac{\partial w}{\partial x} = \frac{D}{G_x h}\left[\frac{\partial^2\beta_x}{\partial x^2} + \frac{(1+\nu)}{2}\frac{\partial^2\beta_y}{\partial x\,\partial y} + \frac{(1-\nu)}{2}\frac{\partial^2\beta_x}{\partial y^2}\right]$$

$$\beta_x - \frac{\partial w}{\partial y} = \frac{D}{G_y h}\left[\frac{\partial^2\beta_y}{\partial y^2} + \frac{(1+\nu)}{2}\frac{\partial^2\beta_x}{\partial x\,\partial y} + \frac{(1-\nu)}{2}\frac{\partial^2\beta_y}{\partial x^2}\right]. \tag{28}$$

Substituting for Q_x and Q_y from eqn (27) into eqn (24), the equilibrium equation for transverse normal forces becomes:

$$D\left[\frac{\partial^3\beta_x}{\partial x^3} + \frac{\partial^3\beta_y}{\partial x^2\,\partial y} + \frac{\partial^3\beta_x}{\partial x\,\partial y^2} + \frac{\partial^3\beta_y}{\partial y^3}\right] = \frac{\partial^2 F}{\partial y^2}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w_0}{\partial x^2}\right) - 2\frac{\partial^2 F}{\partial x\,\partial y}\left(\frac{\partial^2 w}{\partial x\,\partial y} + \frac{\partial^2 w_0}{\partial x\,\partial y}\right) + \frac{\partial^2 F}{\partial x^2}\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w_0}{\partial y^2} - \frac{1}{R}\right). \tag{29}$$

Compatibility equation

After eliminating u and v from the median surface strain–displacement relations [eqn (9)], an equation for the lateral deflection w in terms of the median surface strains ε_x^0, ε_y^0 and γ_{xy}^0 is obtained:

$$\frac{\partial^2\varepsilon_x^0}{\partial y^2} + \frac{\partial^2\varepsilon_y^0}{\partial x^2} - \frac{\partial^2\gamma_{xy}^0}{\partial x\,\partial y} = \frac{1}{R}\frac{\partial^2 w}{\partial x^2} + \left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - 2\frac{\partial^2 w}{\partial x\,\partial y}\frac{\partial^2 w_0}{\partial x\,\partial y} + \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w_0}{\partial y^2}\right). \tag{30}$$

The median surface strains can of course be replaced by the stress resultants using the following Hooke's law relations:

$$\varepsilon_x^0 = \frac{1}{A}(N_x - \nu N_y), \quad \varepsilon_y^0 = \frac{1}{A}(N_y - \nu N_x), \quad \gamma_{xy}^0 = \frac{2(1+\nu)}{A}N_{xy}. \tag{31}$$

Hence one can derive the final form of the compatibility equation in terms of the Airy stress function [see eqn (26)] and the lateral deflection w:

$$\frac{1}{A}\nabla^4 F = \frac{1}{R}\frac{\partial^2 w}{\partial x^2} + \left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - 2\frac{\partial^2 w}{\partial x\,\partial y}\frac{\partial^2 \dot{w}_0}{\partial x\,\partial y} + \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w_0}{\partial y^2}\right). \tag{32}$$

SPECIAL CASES

(a) *Isotropic core* ($G_x = G_y$)

When the core is isotropic, the equilibrium equation for transverse normal forces can be expressed in terms of w and F only. From eqn (17),

$$\beta_x = \frac{\partial w}{\partial x} - \frac{Q_x}{Gh}, \quad \beta_y = \frac{\partial w}{\partial y} - \frac{Q_y}{Gh}.$$

Substituting for β_x and β_y into the equilibrium equation [eqn (29)] yields,

$$D\nabla^4 w - \frac{D}{Gh}\nabla^2\left(\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y}\right) = \frac{\partial^2 F}{\partial y^2}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w_0}{\partial x^2}\right) - 2\frac{\partial^2 F}{\partial x\,\partial y}\left(\frac{\partial^2 w}{\partial x\,\partial y} + \frac{\partial^2 w_0}{\partial x\,\partial y}\right) + \frac{\partial^2 F}{\partial x^2}\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w_0}{\partial y^2} - \frac{1}{R}\right). \quad (33)$$

Eliminating the term $(\partial Q_x/\partial x) + (\partial Q_y/\partial y)$ from eqn (33) and eqn (24) gives:

$$D\nabla^4 w = \left(1 - \frac{D}{Gh}\nabla^2\right)\left[\frac{\partial^2 F}{\partial y^2}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w_0}{\partial x^2}\right) - 2\frac{\partial^2 F}{\partial x\,\partial y}\left(\frac{\partial^2 w}{\partial x\,\partial y} + \frac{\partial^2 w_0}{\partial x\,\partial y}\right) + \frac{\partial^2 F}{\partial x^2}\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w_0}{\partial y^2} - \frac{1}{R}\right)\right]. \quad (34)$$

The above equilibrium equation together with the compatibility equation, eqn (32), are the two governing equations in terms of the two unknowns w and F for sandwich cylinders with isotropic cores.

(b) *Non-shear deformable core* (G = ∞)

The equilibrium equation for sandwich cylinders with a non-shear deformable core can be immediately obtained by setting the core shear modulus G to infinity, i.e.

$$D\nabla^4 w = \frac{\partial^2 F}{\partial y^2}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w_0}{\partial x^2}\right) - 2\frac{\partial^2 F}{\partial x\,\partial y}\left(\frac{\partial^2 w}{\partial x\,\partial y} + \frac{\partial^2 w_0}{\partial x\,\partial y}\right) + \frac{\partial^2 F}{\partial x^2}\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w_0}{\partial y^2} - \frac{1}{R}\right). \quad (35)$$

The above equation and the compatibility equation, eqn (32), are exactly the same as those for homogeneous isotropic shells except for the stiffness terms A and D. Thus all previous analytical results for isotropic cylinders can be immediately applied to sandwich cylinders with non-shear deformable cores when the appropriate in-plane stiffness A and bending stiffness D terms are used.

(c) *Isotropic cylinder—no core* (c = 0)

By setting $c = 0$,

$$A = Et \quad \text{and} \quad D = \frac{Et^3}{12(1-\nu^2)}$$

where $t = t_1 + t_2$, and the previous equilibrium and compatibility equations reduce to the isotropic cylinder case.

BUCKLING ANALYSIS

Prebuckling axisymmetric solution

Assuming an initial axisymmetric shape imperfection, one can write the prebuckling lateral deflection function as

$$w(x, y) = w^*(x)$$

where the asterisk denotes the prebuckling solution.

The solution for the Airy stress function in the prebuckled state is of the following form:

$$F(x, y) = -\tfrac{1}{2}N_0 y^2 + F^*(x) \tag{36}$$

where N_0 is the axially applied load per unit run at the edge.

For the axisymmetric solution,

$$\beta_x(x, y) = \beta_x^*(x) \quad \text{and } \beta_y(x, y) = 0. \tag{37}$$

After substituting these equations into eqns (28), (29) and (32), the four nonlinear governing equations reduce to three linear equations in F^*, w^* and β_x^* for the prebuckling state, i.e.

$$\frac{1}{A}\frac{\partial^4 F^*}{\partial x^4} = \frac{1}{R}\frac{\partial^2 w^*}{\partial x^2}, \quad \beta_x^* - \frac{\partial w^*}{\partial x} = \frac{D}{G_x h}\frac{\partial^2 \beta_x^*}{\partial x^2}, \quad D\frac{\partial^3 \beta_x^*}{\partial x^3} = -N_0\left(\frac{\partial^2 w^*}{\partial x^2} + \frac{\partial^2 w_0}{\partial x^2}\right) - \frac{1}{R}\frac{\partial^2 F^*}{\partial x^2}. \tag{38}$$

The initial axisymmetric shape imperfection can be written as,

$$w_0(x) = -\mu \cos\left(\frac{2\pi x}{l_x}\right). \tag{39}$$

The order of magnitude of the imperfection amplitude μ relative to the imperfection wavelength l_x, as required by eqn (8), is given by

$$\frac{4\mu\pi^2}{l_x^2} \leqslant 0(1). \tag{40}$$

From physical considerations, a particular solution is taken to be in the form:

$$F^*(x) = N\cos\left(\frac{2\pi x}{l_x}\right), \quad w^*(x) = K + M\cos\left(\frac{2\pi x}{l_x}\right), \quad \beta^*(x) = P\sin\left(\frac{2\pi x}{l_x}\right). \tag{41}$$

Substituting the above assumed solution into three linear differential equations, eqn (38), results in three linear algebraic equations from which N, M and P can be easily solved, i.e.

$$N = \frac{-\mu\lambda\alpha}{2\rho^2(\lambda_0 - \lambda)}, \quad M = \frac{-\mu\lambda}{(\lambda_0 - \lambda)}, \quad P = \frac{\mu\lambda\pi}{(1 + 2\chi_x\rho^2)(\lambda_0 - \lambda)l_x} \tag{42}$$

where

$$\alpha = (AD)^{1/2}, \quad \gamma = \left(\frac{A}{D}\right)^{1/2}, \quad \chi_x = \frac{\alpha}{RhG_x}$$

$$\rho = \frac{l}{\sqrt{2}l_x}, \quad l = 2\pi\left(\frac{R}{\gamma}\right)^{1/2}, \quad \lambda_0 = \frac{1}{4\rho^2} + \frac{\rho^2}{1 + 2\chi_x\rho^2}$$

and

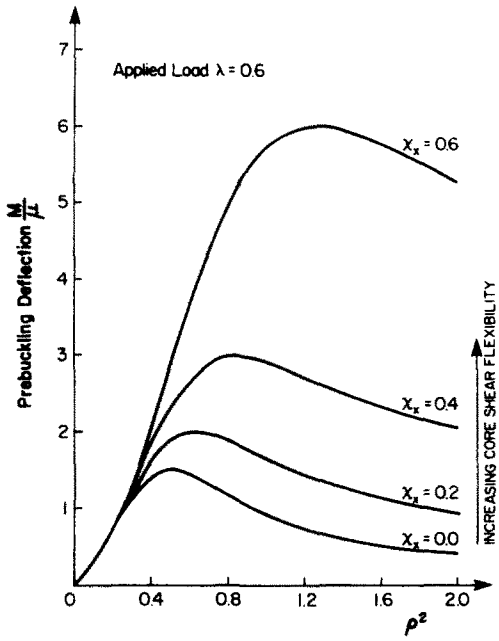

Fig. 3. Prebuckling axisymmetric lateral deflection as a function of axial shear core flexibility.

$$\lambda = \frac{N_0 R}{2\alpha}. \tag{43}$$

Axisymmetric buckling of a perfect cylinder

An interesting consequence of eqn (42) is that it is possible to obtain an axisymmetric buckling solution for the perfect shell at this stage of the analysis. For the case of a perfect shell, μ vanishes and for non-zero values of M, the deflection mode as specified by eqn (41) then corresponds to the axisymmetric buckling mode of the perfect shell. The condition for non-zero values of M requires the denominator to vanish. Thus, $\lambda = \lambda_0$ and the axisymmetric buckling coefficient (λ_a) for the perfect shell is obtained by minimizing λ_0 with respect to the axial wave number ρ. The equation $\partial\lambda_0/\partial\rho^2 = 0$ has two real solutions, namely:

$$\rho^2 = \frac{1}{2(1-\chi_x)}, \quad \rho^2 = \infty. \tag{44}$$

When the axial core shear flexibility coefficient $\chi_x \geqslant 1$, $\rho^2 = \infty$ is the only real root. Substituting eqn (44) into the equation for λ_0 yields the axisymmetric buckling coefficient λ_a for the perfect cylinder:

$$\lambda_a = 1 - \frac{\chi_x}{2} \quad \text{when } \chi_x \leqslant 1$$

$$\lambda_a = \frac{1}{2\chi_x} \quad \text{when } \chi_x > 1. \tag{45}$$

Growth of prebuckling lateral deflections

The amplitude of the lateral deflection M grows under increasing load. Of engineering interest is the imperfection wave number which gives the largest deflection for a given load. From eqn (42), the value of M becomes a maximum, for a given load λ and imperfection amplitude μ, when λ_0 is a minimum. As noted earlier, the axial wave number which minimizes λ_0 is the axisymmetric buckling wave number for the perfect shell. Thus, imperfections in the shape of the classical axisymmetric buckling mode will yield the largest prebuckling deflection. Figure 3 illustrates the prebuckling deflection for various axial wave

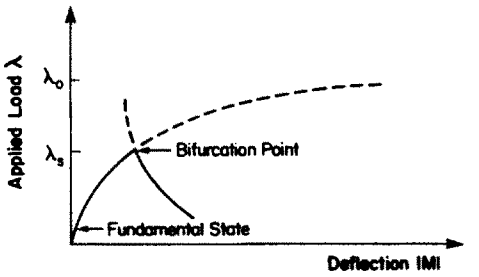

Fig. 4. Bifurcation from fundamental equilibrium path.

numbers at a given load. It is also evident that the prebuckling deflection increases with increasing axial core shear flexibility.

Bifurcation buckling solution

Under increasing load the amplitude of the lateral deflection M will grow in a hyperbolic fashion until the bifurcation point is reached, where there is an intersection of equilibrium paths (Fig. 4). Let us define the terms $\bar{w}(x,y), f(x,y), b_x(x,y)$ and $b_y(x,y)$ associated with bifurcation from the fundamental axisymmetric state. Hence, one can write,

$$F(x,y) = -\tfrac{1}{2}N_0 y^2 + F^*(x) + f(x,y), \quad w(x,y) = w^*(x) + \bar{w}(x,y)$$
$$\beta_x(x,y) = \beta_x^*(x) + b_x(x,y), \quad \beta_y(x,y) = b_y(x,y). \tag{46}$$

The bifurcation point can be determined by considering the existence of a second solution to the governing equations which is infinitesimally close to the fundamental solution. With $\bar{w}(x,y), f(x,y), b_x(x,y)$ and $b_y(x,y)$ representing infinitesimal deviations from the fundamental state, after substituting eqn (46) into the governing equations [eqns (28), (29), (32)], the resulting equations can be linearized with respect to $w(x,y)$ and $f(x,y)$ to obtain the linearized equations of neutral equilibrium:

Compatibility equation:

$$\nabla^4 f = \frac{A}{R}\left[\frac{\partial^2 \bar{w}}{\partial x^2} - \frac{2\gamma\rho^2\lambda_0}{(\lambda_0-\lambda)}\mu\cos\left(\frac{2\pi x}{l_x}\right)\frac{\partial^2 \bar{w}}{\partial y^2}\right]. \tag{47}$$

Equilibrium equations:

$$b_x - \frac{\partial \bar{w}}{\partial y} = \frac{D}{G_x h}\left[\frac{\partial^2 b_x}{\partial x^2} + \frac{(1+\nu)}{2}\frac{\partial^2 b_y}{\partial x\,\partial y} + \frac{(1-\nu)}{2}\frac{\partial^2 b_x}{\partial y^2}\right]$$

$$b_y - \frac{\partial \bar{w}}{\partial x} = \frac{D}{G_y h}\left[\frac{\partial^2 b_y}{\partial y^2} + \frac{(1+\nu)}{2}\frac{\partial^2 b_x}{\partial x\,\partial y} + \frac{(1-\nu)}{2}\frac{\partial^2 b_y}{\partial x^2}\right]$$

$$D\left[\frac{\partial^3 b_x}{\partial x^3} + \frac{\partial^3 b_y}{\partial x^2\,\partial y} + \frac{\partial^3 b_x}{\partial x\,\partial y^2} + \frac{\partial^3 b_y}{\partial y^3}\right] = -\frac{2\alpha\lambda}{R}\frac{\partial^2 \bar{w}}{\partial x^2}$$
$$+ \frac{2\gamma\lambda\rho^2\mu}{(\lambda_0-\lambda)R}\cos\left(\frac{2\pi x}{l_x}\right)\frac{\partial^2 f}{\partial y^2} - \frac{\gamma\alpha\lambda\mu}{(\lambda_0-\lambda)R}\cos\left(\frac{2\pi x}{l_x}\right)\frac{\partial^2 \bar{w}}{\partial y^2} - \frac{1}{R}\frac{\partial^2 f}{\partial x^2}. \tag{48}$$

A rigorous solution of the above coupled equations of neutral equilibrium with given boundary conditions is extremely difficult. However, an approximate solution can be obtained using an assumed mode of the form,

$$\bar{w}(x,y) = \sum_{i=0}^{\infty} K_i \cos\frac{(2i+1)\pi x}{l_x}\cos\frac{\pi y}{l_y} \tag{49}$$

where

$$K_i = 0 \quad \text{for } i < 0.$$

Substituting the assumed mode into the compatibility [eqn (47)] and equilibrium equations [eqn (46)], exact particular solutions for f, b_x and b_y are obtained in terms of the coefficients K_i, i.e.

$$f(x,y) = \sum_{i=-1}^{\infty} H_i \cos\frac{(2i+1)\pi x}{l_x}\cos\frac{\pi y}{l_y}, \quad b_x = \sum_{i=0}^{\infty} A_i \sin\frac{(2i+1)\pi x}{l_x}\sin\frac{\pi y}{l_y},$$

$$b_y = \sum_{i=0}^{\infty} B_i \cos\frac{(2i+1)\pi x}{l_x}\sin\frac{\pi y}{l_y} \tag{50}$$

where

$$H_i = \frac{-2\alpha\left[(2i+1)^2\rho^2 K_i - \dfrac{\mu\gamma\rho^2\tau^2\lambda_0}{(\lambda_0-\lambda)}(K_{i-1}+K_{i+1})\right]}{[(2i+1)^2\rho^2+\tau^2]^2}$$

$$A_i = \frac{-K_i\left[\dfrac{a_{2i}(2i+1)\pi}{l_x} - \dfrac{b_{1i}\pi}{l_y}\right]}{(a_{1i}a_{2i}-b_{1i}b_{2i})}, \quad B_i = \frac{-K_i\left[\dfrac{a_{1i}\pi}{l_y} - \dfrac{b_{2i}(2i+1)\pi}{l_x}\right]}{(a_{1i}a_{2i}-b_{1i}b_{2i})}$$

$$a_{1i} = 1+\frac{\chi_x}{2}\left[(2i+1)^2\rho^2+\frac{(1-\nu)}{2}\tau^2\right], \quad a_{2i} = 1+\frac{\chi_y}{2}\left[\tau^2+\frac{(1-\nu)}{2}(2i+1)^2\rho^2\right]$$

$$b_{1i} = \chi_x\frac{(1+\nu)(2i+1)}{4}\rho\tau, \quad b_{2i} = \chi_y\frac{(1+\nu)(2i+1)}{4}\rho\tau. \tag{51}$$

Since eqn (50) satisfies the first three governing equations exactly, Galerkin's procedure can then be applied to the remaining equilibrium equation to obtain an approximate solution for the buckling stress. Substituting eqn (50) into the last equilibrium equation yields the following error function:

$$\xi(x,y) = \frac{A}{(2R)^2}\sum_{i=-2}^{\infty} S_i \cos\frac{(2i+1)\pi x}{l_x}\cos\frac{\pi y}{l_y}. \tag{52}$$

In the Galerkin procedure, the error function $\xi(x,y)$ is multiplied by

$$\cos\frac{(2j+1)\pi x}{l_x}\cdot\cos\frac{\pi y}{l_y} \quad \text{for } j = 0,1,2,\ldots,$$

integrated over the whole shell and equated to zero. Thus,

$$\int\int \xi(x,y)\cos\frac{(2j+1)\pi x}{l_x}\cos\frac{\pi y}{l_y}\,dx\,dy = 0 \tag{53}$$

yields an infinite system of linear algebraic equations for the coefficients K_i in the following form:

$$S_{-1}+S_0=0,\quad S_{-2}+S_1=0,\quad S_j=0\quad \text{for } j\geqslant 3 \tag{54}$$

where

$$S_i = U_{i-1}K_{i-2}+V_iK_{i-1}+(W_i+U_{i-1}+U_{i+1})K_i+V_{i+1}K_{i+1}+U_{i+1}K_{i+2}$$

$$U_i = \frac{(2\mu\gamma\rho^2\tau^2\lambda_0)^2}{(\lambda_0-\lambda)^2}\cdot\frac{1}{[(2j+1)^2\rho^2+\tau^2]}$$

$$V_i = \frac{-\mu\gamma\tau^2\lambda}{(\lambda_0-\lambda)}-\frac{4\mu\gamma\rho^2\tau^2\lambda_0}{(\lambda_0-\lambda)}\left\{\frac{(2i-1)^2\rho^2}{[(2i-1)^2\rho^2+\tau^2]^2}+\frac{(2i+1)^2\rho^2}{[(2i+1)^2\rho^2+\tau^2]^2}\right\}$$

$$W_i = k_i-4(2i+1)^2\rho^2\chi+\frac{4(2i+1)^4\rho^4}{[(2i+1)^2\rho^2+\tau^2]^2}$$

$$k_i = [a_{2i}(2i+1)^4\rho^4-(b_{1i}+b_{2i})(2i+1)^3\rho^3\tau+(a_{1i}+a_{2i})(2i+1)^2\rho^2\tau^2 -(b_{1i}+b_{2i})(2i+1)\rho\tau^3+a_{1i}\tau^4]/(a_{1i}a_{2i}-b_{1i}b_{2i}). \tag{55}$$

ANALYTICAL RESULTS

Sandwich cylinders can be roughly classified into three categories according to their core shear flexibility:

(a) stiff core sandwich cylinders with χ_x, $\chi_y < 0.1$;

(b) moderately flexible core sandwich cylinders with $0.1 \leqslant \chi_x$ or $0.1 \leqslant \chi_y$, and χ_x, $\chi_y < 0.9$;

(c) weak core sandwich cylinders with χ_x or $\chi_y > 0.9$.

Because of the assumption that the facings' flexural stiffness about their middle surfaces can be neglected, the results presented for weak core cylinders are not expected to be accurate.

First term approximate solution

Using only the first term

$$K_0\cos\frac{\pi x}{l_x}\cdot\cos\frac{\pi y}{l_y}$$

in the assumed buckling mode [eqn (49)], the first equation from the set of equations [eqn (54)] is obtained. For a non-trivial solution, $K_0 \neq 0$ gives:

$$v_0+w_0+u_{-1}+u_1=0 \tag{56}$$

or

$$k_0-4\rho^2\lambda+\frac{4\rho^4}{(\rho^2+\tau^2)^2}-\frac{\mu\gamma\tau^2\lambda}{(\lambda_0-\lambda)}-\frac{8\mu\gamma\rho^4\tau^2\lambda_0}{(\lambda_0-\lambda)(\rho^2+\tau^2)^2} +\frac{(2\mu\gamma\rho^2\tau^2\lambda_0)^2}{(\lambda_0-\lambda)^2}\left[\frac{1}{(\rho^2+\tau^2)^2}+\frac{1}{(9\rho^2+\tau^2)^2}\right]=0. \tag{57}$$

The above equation can be written as a cubic polynomial of λ:

$$\lambda^3+P\lambda^2+Q\lambda+R=0 \tag{58}$$

where

$$P = -(2\lambda_0 + \lambda_1 + E_1), \quad Q = \lambda_0^2 + 2\lambda_0\lambda_1 + \lambda_0 E_1 - E_2$$

$$R = -\lambda_1\lambda_0^2 + \lambda_0 E_2 - E_3, \quad \lambda_1 = \frac{k_0}{4\rho^2} + \frac{\rho^2}{(\rho^2+\tau^2)^2}$$

$$E_1 = \frac{\mu\lambda\tau^2}{4\rho^2}, \quad E_2 = \frac{2\mu\gamma\rho^2\tau^2\lambda_0}{(\rho^2+\tau^2)^2}, \quad E_3 = (\mu\gamma\rho\tau^2\lambda_0)^2\left[\frac{1}{(\rho^2+\tau^2)^2} + \frac{1}{(9\rho^2+\tau^2)^2}\right]. \tag{59}$$

Perfect cylinders

The buckling load for a geometrically perfect sandwich cylinder can be easily obtained by setting $\mu = 0$ in eqn (57), i.e.

$$\lambda = \frac{k_0}{4\rho^2} + \frac{\rho^2}{(\rho^2+\tau^2)^2}. \tag{60}$$

Of particular interest is the case of axisymmetric buckling of the perfect cylinder. Thus, setting $\tau = 0$, the resulting expression for λ is minimized with respect to ρ, to give:

$$\lambda_a = 1 - \frac{\chi_x}{2} \quad \text{with } \rho^2 = \frac{2}{(1-\chi_x)} \quad \text{for } \chi_x < 1$$

$$\lambda_a = \frac{1}{2\chi_x} \quad \text{with } \rho^2 = \infty \quad \text{for } \chi_x \geqslant 1. \tag{61}$$

This of course is the same result as that obtained in eqn (45) except that ρ differs by a factor of 2 due to the difference in the definition of the buckling modes.

Because of the orthotropy of the core, the expression for k_0 is complicated [see eqn (55)]. However it simplifies considerably for the case of an isotropic core and the corresponding buckling stress is,

$$\lambda = \frac{(\rho^2+\tau^2)^2}{4\rho^2\left[1 + \frac{\chi}{12}(\rho^2+\tau^2)\right]} + \frac{\rho^2}{(\rho^2+\tau^2)^2}. \tag{62}$$

As shown by Zahn and Kuenzi (1963), perfect sandwich cylinders with isotropic cores buckle in the axisymmetric mode and thus the buckling coefficient is given by eqn (61). In the limiting case of a non-shear deformable core eqn (62) reduces to,

$$\lambda = \frac{(\rho^2+\tau^2)^2}{4\rho^2} + \frac{\rho^2}{(\rho^2+\tau^2)^2}. \tag{63}$$

This is the same expression as that for an isotropic cylinder and the buckling load is obtained by minimizing λ. Any combination of ρ and τ that satisfies

$$(\rho^2+\tau^2)^2 + 2\rho^2 \tag{64}$$

will yield a minimum $\lambda = 1$.

As mentioned earlier, because of the complexity of the expression for the case of an orthotropic core, it is not easy to analytically minimize λ with respect to ρ and τ. Consequently, numerical minimization was carried out to evaluate the "classical" buckling stress for sandwich cylinders for varying core shear flexibility coefficients χ_x, χ_y, as shown in Figs

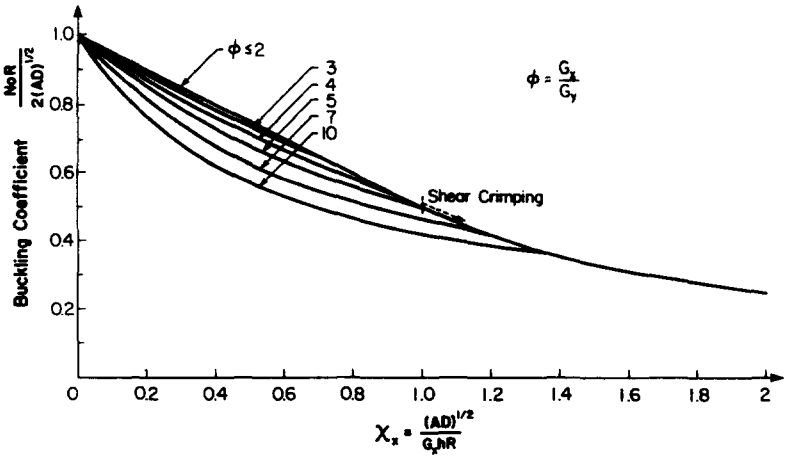

Fig. 5. Effect of orthotropic core shear stiffness ratio (ϕ) on compressive buckling strength of sandwich cylinders with isotropic facings.

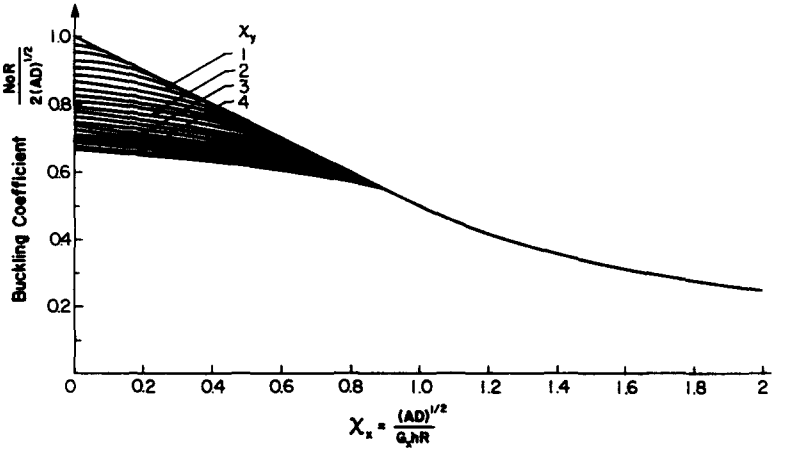

Fig. 6. Buckling coefficient for perfect sandwich cylinder with orthotropic core.

5 and 6. Results of Zahn and Kuenzi (1963) are reproduced in the case of $\phi = 5$, where $\phi = \chi_y/\chi_x = G_x/G_y$. For large values of ϕ (e.g. 7, 10), the plot in Fig. 5 shows that it is possible to have a buckling load lower than the shear crimping† load (Zahn and Kuenzi, 1963) for $\chi_x \geq 1$ in a sandwich cylinder with an orthotropic core. Numerical results indicated that for $\phi \leq 2.0$ the sandwich cylinder buckles in the axisymmetric mode and thus the buckling coefficient is given by the simple expressions in eqn (61). For design purposes, buckling load and stress can be calculated using the following relations:

Buckling load:

$$P_c = 4\pi\alpha\lambda_c = 2\pi R N_c$$

Buckling stress:

$$\sigma_c = \frac{N_c}{(t_1+t_2)} = \frac{E_f t_e}{R\sqrt{3(1-\nu^2)}} \tag{65}$$

where t_e = the "equivalent thickness", which is defined as the thickness of an "equivalent" isotropic cylinder having the same ratio of in-plane to bending stiffness as the sandwich shell, i.e. from eqn (43),

† Special form of general instability having short buckle wavelength, due to low transverse shear modulus of core (Sullins *et al.*, 1969).

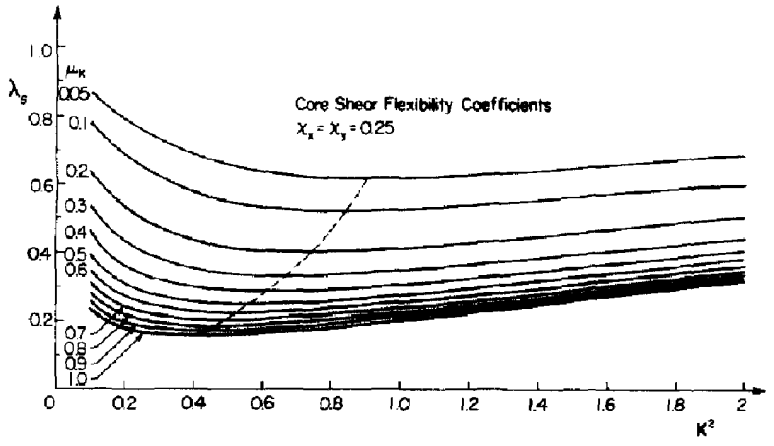

Fig. 7. Effect of imperfection amplitude on λ_s vs K^2 for isotropic cores.

$$\gamma^2 = \left(\frac{A}{D}\right)_{isotropic} = \frac{12(1-\nu^2)}{t_e^2}$$

or

$$t_e = \frac{2\sqrt{3(1-\nu^2)}}{\gamma}. \tag{66}$$

Imperfect cylinders

The cubic eigenvalue eqn (58) yields the bifurcation load coefficient λ_s for an axisymmetric imperfect sandwich cylinder by minimizing λ with respect to the circumferential wave number τ for a given imperfection wave number ρ. Because of the complexity of the equation, the minimization is done numerically and the smallest real root of the cubic eigenvalue equation is selected.

For the case of a non-shear deformable core ($\chi_x = \chi_y = 0$), the cubic eigenvalue equation reduces to that for an isotropic cylinder when the nondimensional imperfection parameter is taken in the form $\mu_k = \mu/t_e$. Thus the analytical results for isotropic shells can be applied to "stiff-core" sandwich cylinders.

The next class of cylinders considered is that for isotropic cores ($\chi_x = \chi_y$). Figures 7, 8 and 9 are plots of the bifurcation load coefficient λ_s versus imperfection amplitude (μ_k) and the wave number parameter $K^2 = \rho^2/\rho_c^2$ where $\rho_c^2 = 1/[2(1-\chi_x)]$ is the classical axisymmetric buckling mode wave number. It can be seen that there exists a critical axial wave number where the degradation is largest for a given imperfection amplitude. The dashed

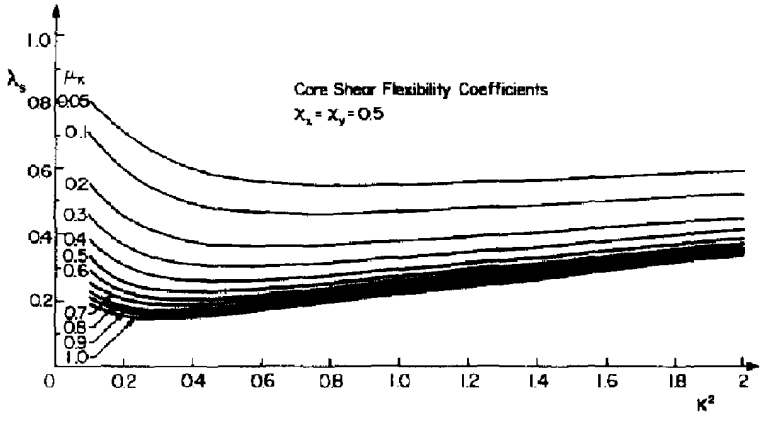

Fig. 8. Effect of imperfection amplitude μ_K on λ_s vs K^2 for isotropic cores.

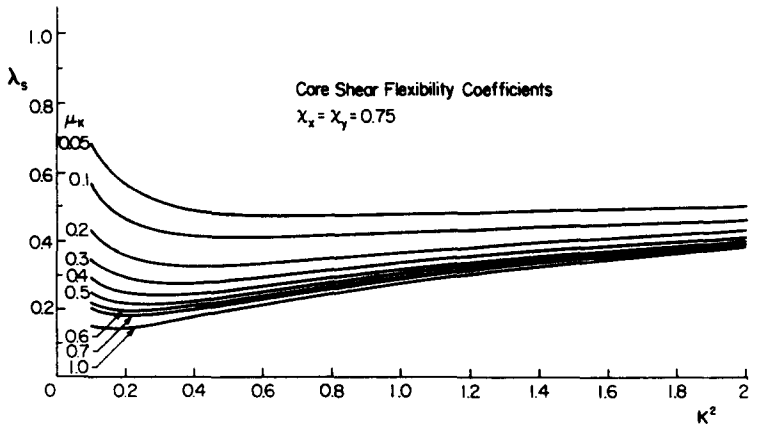

Fig. 9. Effect of imperfection amplitude μ_K on λ_s vs K^2 for isotropic cores.

line in Fig. 7 joins the critical wave number for each imperfection amplitude. The critical wave number decreases with increasing imperfection. From these three figures it can also be seen that critical K^2 decreases with increasing core shear flexibility. Figure 10 demonstrates that imperfections can lead to buckling loads below the shear crimping value of the "perfect" cylinder, although these results maybe somewhat inaccurate for $\chi_x > 0.9$.

For the case of orthotropic cores, it is apparent from Fig. 12 that the circumferential core shear flexibility coefficient χ_y has little effect on the critical buckling mode wave number

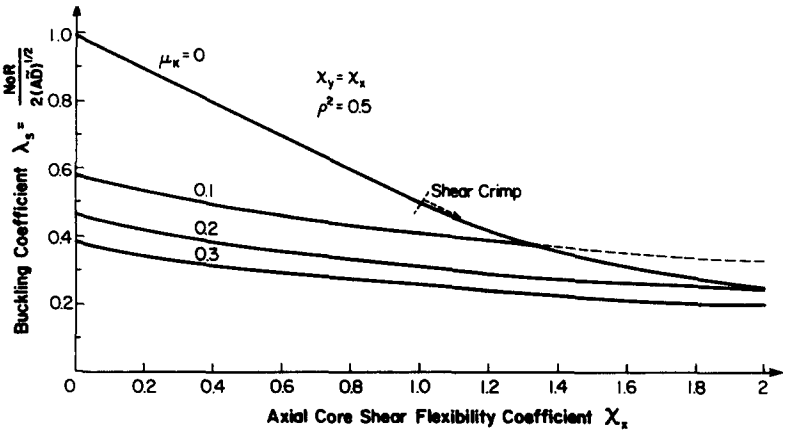

Fig. 10. Effect of imperfection amplitude on λ_s vs χ_x for isotropic cores.

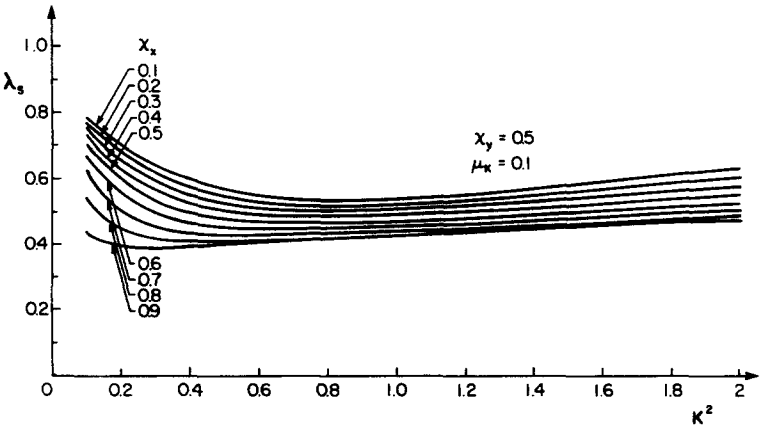

Fig. 11. Effect of χ_x on imperfect orthotropic honeycomb sandwich cylinders.

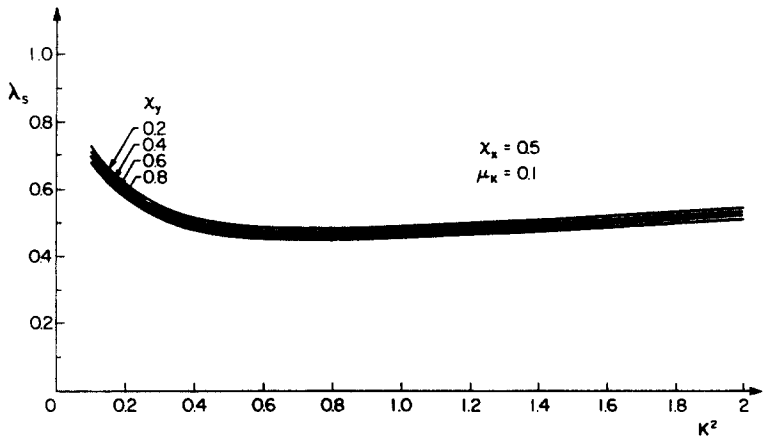

Fig. 12. Effect of χ_y on imperfect orthotropic honeycomb sandwich cylinders.

(K^2). However, Fig. 11 reveals that the critical K^2 parameter decreases with increasing axial core shear flexibility. In general it can be noted that increasing core shear flexibility χ_x, χ_y decreases the buckling strength.

From an engineering design viewpoint, it is useful to plot a "knock-down" factor corresponding to the minimum buckling load (λ_m) associated with the critical wave number, as a function of the non-dimensional imperfection parameter μ_k. Figures 13 and 14 present

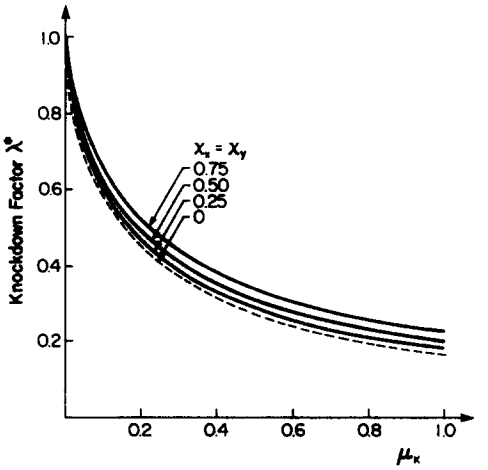

Fig. 13. Effect of core shear flexibility coefficient on λ^* vs μ_K for isotropic core.

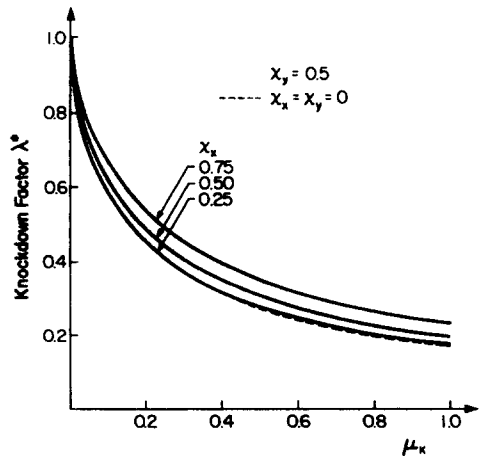

Fig. 14. Effect of axial core shear flexibility (χ_x) on λ^* vs μ_K for orthotropic core.

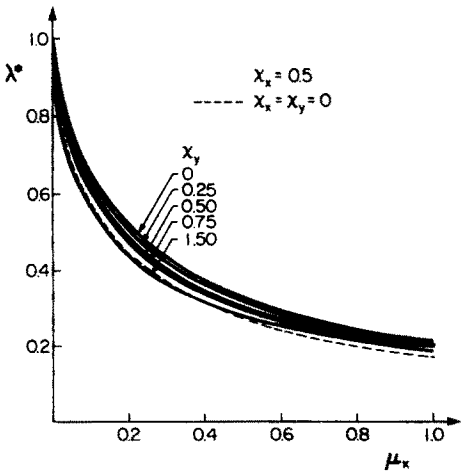

Fig. 15. Effect of axisymmetric shape imperfections and circumferential core shear flexibility (χ_y) on compressive buckling strength for orthotropic cores.

such plots for isotropic and orthotropic cores, respectively. The "knock-down" factor is defined as $\lambda^* = \lambda_m/\lambda_c$, where λ_c is the perfect cylinder buckling coefficient, as determined from eqn (60). Again it is evident that decreasing the core flexibility coefficients decreases λ^*. However, it is interesting to note from Fig. 15, that for a fixed value of $\chi_x = 0.5$, increasing the circumferential core flexibility coefficient χ_y actually decreases λ^*. In each of these three graphs, the non-shear deformable core case is shown, which corresponds to the isotropic cylinder solution as discussed earlier. In most instances, it provides a lower bound curve for the solutions obtained.

CONCLUSIONS

Solutions have been obtained for axisymmetric imperfect sandwich cylinders having isotropic facings and orthotropic shear deformable cores. Shear core flexibility has been shown to have a significant effect on the buckling strength, both for the perfect and imperfect cylinders. For very stiff cores, buckling behaviour is very similar to that of an isotropic shell based on an "equivalent" thickness formulation. Finally, it is worth noting that the "knock-down" curves presented can be used to assist the design engineer in calculating the load reductions for sandwich cylinders containing random imperfections following the methodology described in Tennyson *et al.* (1971) providing an estimate of the maximum root mean square imperfection amplitude is known.

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